Student Number:
Teacher:

St George Girls High School

Mathematics Extension 1

2023 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided

  For questions in **Section II:**
  - Answer the questions in the booklets provided
  - Start each question in a new writing booklet
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 70

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9 –15)

- Attempt Questions 11–15
- Allow about 1 hour and 45 minutes for this section

Q1-10	/10
Q11	/13
Q12	/12
Q13	/12
Q14	/12
Q15	/11
TOTAL	**/70**
	%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. What is the range of $y = 4\cos^{-1}\frac{3x}{2}$?

   A. Range: $-4\pi \leq y \leq 4\pi$.

   B. Range: $0 \leq y \leq -4\pi$.

   C. Range: $0 \leq y \leq 4\pi$.

   D. Range: $0 \leq y \leq 4$.

2. Consider the equation $3x^3 + 2x^2 - x + 5 = 0$.
   What is the product of the roots of this polynomial equation?

   A. $\frac{5}{3}$

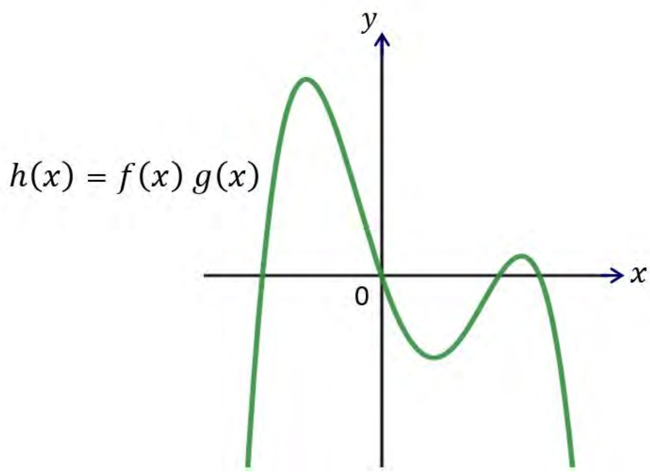
   B. $\frac{1}{3}$

   C. $-\frac{1}{3}$

   D. $-\frac{5}{3}$

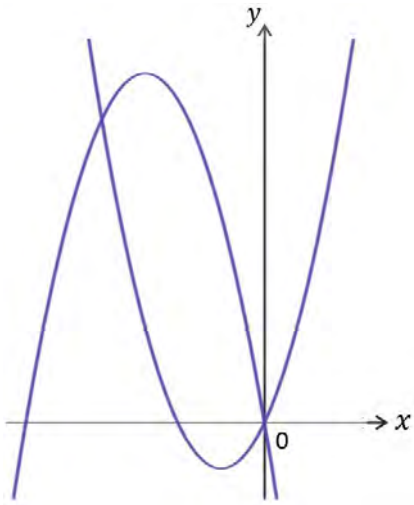
Section I continued

3. The diagram shows the graph of $h(x)$ which is the product of the functions $f(x)$ and $g(x)$.

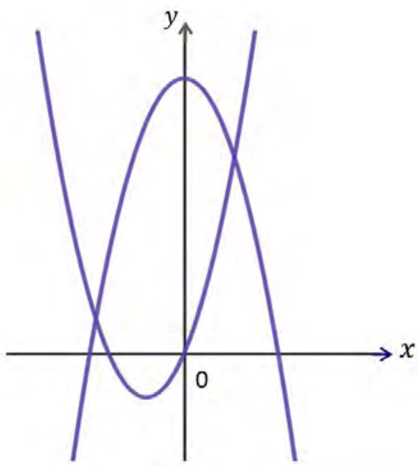


Which of the following is the best option to represent the graphs of the two functions $f(x)$ and $g(x)$?

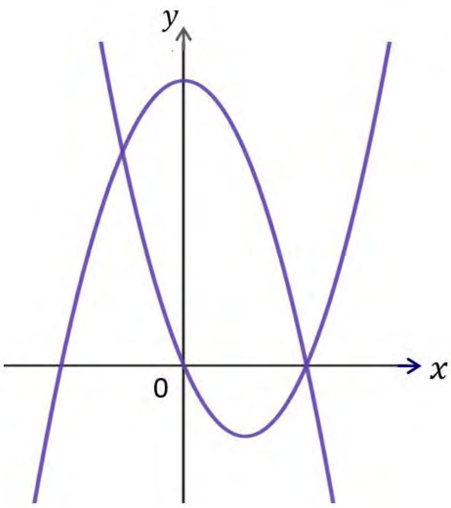
A.



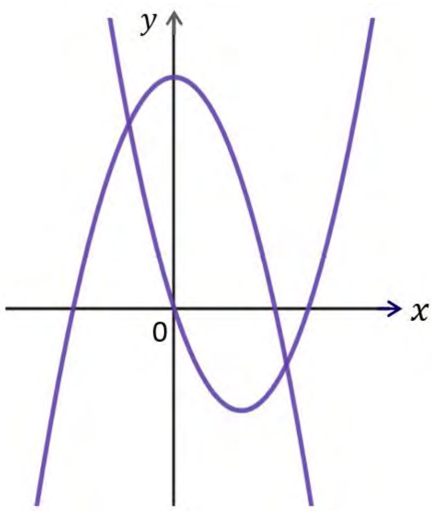
B.



C.



D.



Section I continued

4. Which of the following is equivalent to $\int \cos^2 7x \ dx$?

A. $\frac{x}{2} + \frac{1}{14} \cos 7x + C$

B. $\frac{x}{2} + \frac{1}{28} \sin 14x + C$

C. $\frac{x}{2} + \frac{1}{14} \sin 7x + C$

D. $\frac{x}{2} - \frac{1}{28} \cos 14x + C$

5. Which of the following is the exact value of $\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4}{\sqrt{9-x^2}} dx$?

A. $-\pi$

B. $-\frac{\pi}{4}$

C. $\frac{\pi}{4}$

D. π

6. How many years will it take for a bacterial population to reach 8000 if the population is modelled by the following function?

$$f(t) = \frac{10000}{1 + e^{-0.12(t-20)}}$$, where t represents the number of years.

A. $t = 31.6$

B. $t = 30$

C. $t = \ln 0.12$

D. $t = \ln 0.25$

Section I continued

7. What is the term independent of x in the expansion of $\left(x^3 + \frac{2}{x}\right)^{20}$?

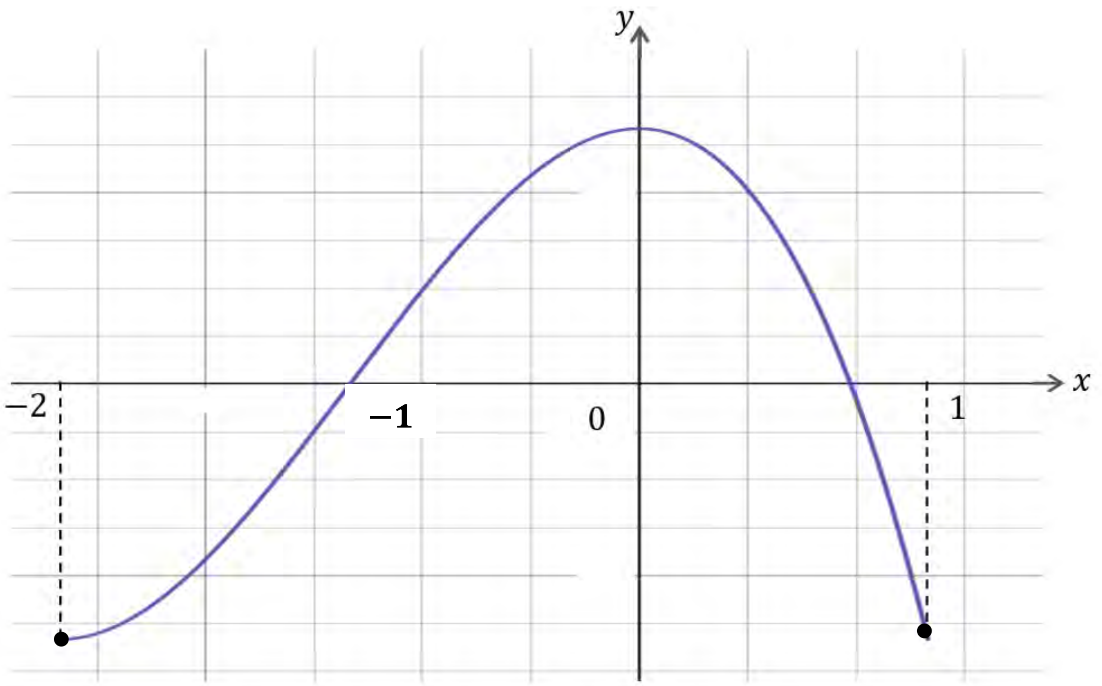
A. $\binom{20}{10} \cdot 2^{10}$

B. $\binom{20}{5} \cdot 2^{15}$

C. $\binom{20}{5} \cdot 2^{25}$

D. $\binom{20}{4} \cdot 2^{16}$

8. A parametric function is graphed below.



Which of the following shows the parametric equations of the curve above for $-1 \le t \le 2$?

A. $x = t - 1$ and $y = 3t - t^3$

B. $x = t + 1$ and $y = 3t - t^3$

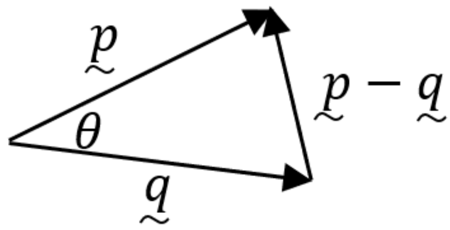
C. $x = t + 1$ and $y = -3t + t^3$

D. $x = t - 1$ and $y = 3t + t^3$

Section I continued

9. The magnitudes of two vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are 3 and 2 respectively.

   The angle between these two vectors is θ such that $\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$.



NOT TO SCALE

Which of the following is the correct range of $|\underset{\sim}{p} - \underset{\sim}{q}|$?

A. $7 \leq |\underset{\sim}{p} - \underset{\sim}{q}| \leq 19$

B. $7 \leq |\underset{\sim}{p} - \underset{\sim}{q}| \leq 13$

C. $\sqrt{7} \leq |\underset{\sim}{p} - \underset{\sim}{q}| \leq \sqrt{19}$

D. $\sqrt{7} \leq |\underset{\sim}{p} - \underset{\sim}{q}| \leq 13$

Section I continued

10. Consider the vector $\underset{\sim}{p} = 2\,(\ln x\,)^2\,\underset{\sim}{i} \;+ 3\ln x\,\underset{\sim}{j}$ and the vector $\underset{\sim}{q} = -5\,\underset{\sim}{i} + 12\,\underset{\sim}{j}$.

The length of the projection of $\underset{\sim}{p}$ onto $\underset{\sim}{q}$, where $\ln x$ is a positive integer, is $\frac{32}{13}$.

What is the value of $|\underset{\sim}{p}|$?

A. 5

B. 10

C. 13

D. 15

End of Section I

<u>Section II</u>
60 marks
Attempt Questions 11 – 15
Allow about 1 hour and 45 minutes for this section

In Questions 11-15, your responses should include relevant mathematical reasoning and/or calculations

Question 11 (13 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Show that $(x^2 + y)^4 = x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$. **1**

(ii) Hence, or otherwise, expand $(b^4 - 1)^4$. **2**

(b) Solve the inequality $\frac{1}{x-3} \geq -1$. **3**

(c) The polynomial $P(x) = 2x^3 + mx^2 + nx - 20$ has a double root and

$P(2) = P'(2) = 0$.

(i) Find the values of m and n. **3**

(ii) Hence, find the other root of $P(x)$. **2**

(d) Express $9 \sin x + 40 \cos x$ in the form $R \sin(x + \alpha)$ where $0 \leq \alpha \leq \frac{\pi}{2}$. **2**
Leave α in terms of 3 significant figures.

Section II continued

Question 12 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the vector $\underset{\sim}{a} = \underset{\sim}{i} - 2\underset{\sim}{j}$ and the vector $\underset{\sim}{b} = x\underset{\sim}{i} + y\underset{\sim}{j}$ where x and y are real. **3**

$\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{a}$ and $|\underset{\sim}{b}| = 2\sqrt{5}$.

Find the possible values of x and y.

(b) Find the equation of the tangent to the curve $y = \cos^{-1} 2x$ at $x = 0$. **3**

(c) Use mathematical induction to prove that, for all integers $n \geq 1$, **3**

$$(2^1 + 2) + (2^2 + 4) + (2^3 + 6) + \ldots + (2^n + 2n) = 2^{n+1} + n(n+1) - 2.$$

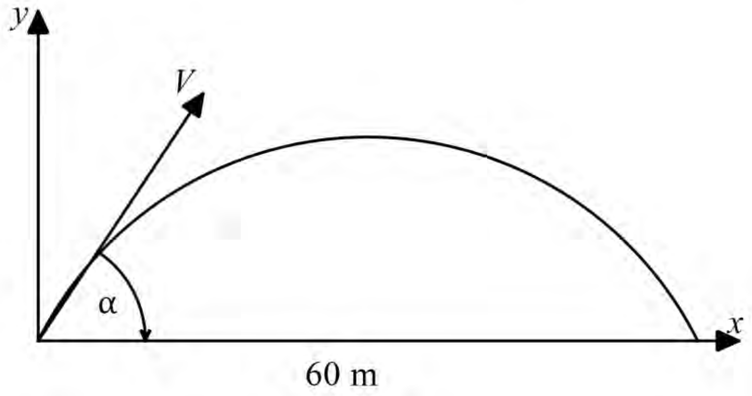
(d) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{3}} \cos 4x \cos 2x \, dx$. **3**

Section II continued

Question 13 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A cricket player hits a ball from ground level. The ball is projected under gravity with speed V ms^{-1} in a direction making an angle α with the horizontal. **3**

The ball lands on the boundary, which is 60 metres away.



The displacement vector of the ball is

$$\underset{\sim}{r}(t) = \begin{pmatrix} Vt\cos\alpha \\ -\frac{1}{2}gt^2 + Vt\sin\alpha \end{pmatrix},$$

where the horizontal and vertical displacement is measured in metres after t seconds and $g = 9.8\ \text{ms}^{-2}$. (DO NOT PROVE THESE)

If the angle of flight to the horizontal is 28° at the instant the ball leaves the bat, calculate the initial speed of the ball.

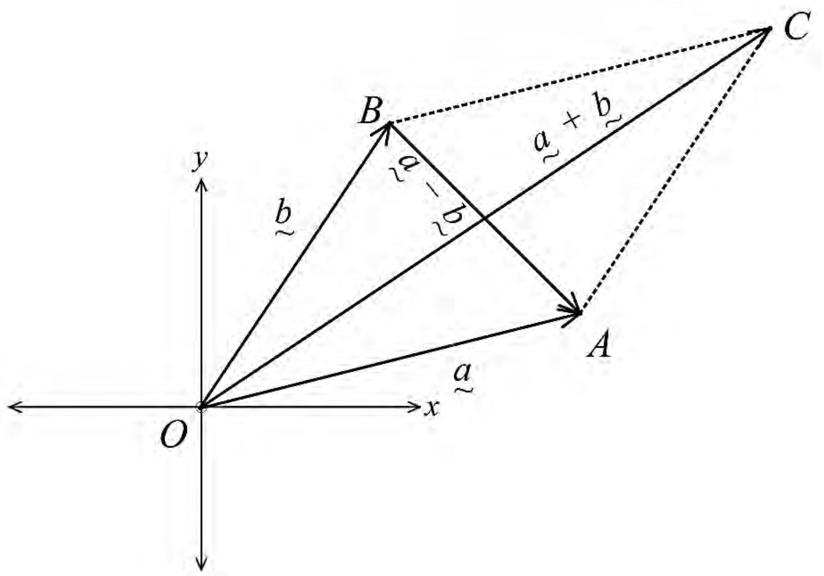
(b) It is given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. (DO NOT prove this.) **3**

Hence, prove that the cubic equation $4x^3 - 3x + 1 = 0$, has only **two** solutions.

Section II continued

(c) The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ form two adjacent sides of a parallelogram $OACB$.

The diagonals of the parallelogram are the vectors $\underset{\sim}{a} + \underset{\sim}{b}$ and $\underset{\sim}{a} - \underset{\sim}{b}$, as shown.



(i) Prove that for the parallelogram, $(\underset{\sim}{a} + \underset{\sim}{b}).(\underset{\sim}{a} - \underset{\sim}{b}) = |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2$. **2**

(ii) Hence, show that for a rhombus, the diagonals are perpendicular. **1**

(d) Harry sets up a playlist which has ten tracks on the media player on his phone. **3**

Three of the tracks are by G Flip and two are by D'Arcy.

The media player is put into "shuffle" mode, where each track is played exactly once, but in a random order."

What is the **probability** that the three tracks by G Flip are played together and the two by D'Arcy are also played together?

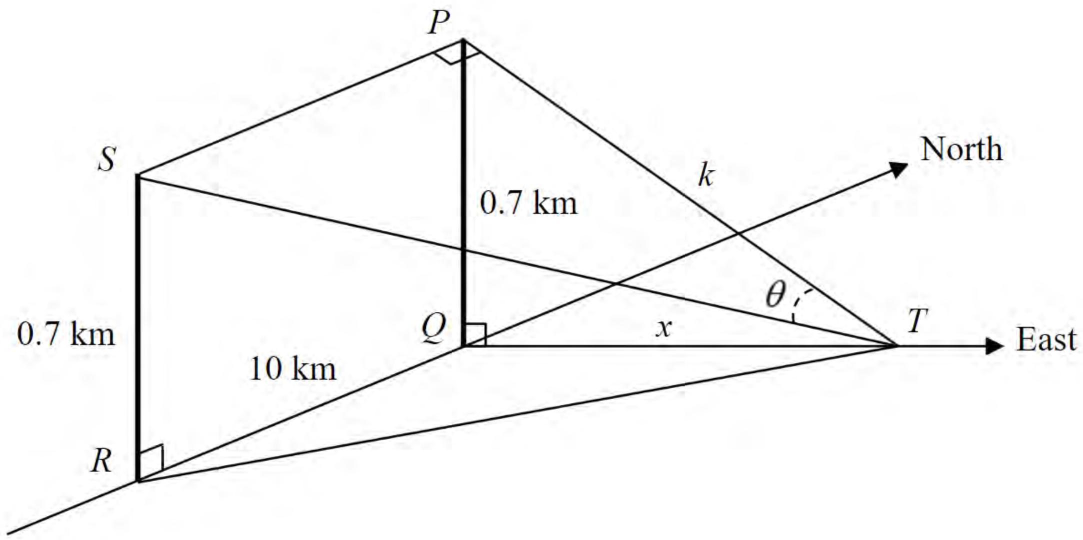
(The five tracks do not necessarily have to be played together)

Section II continued

Question 14 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A committee containing 4 men and 2 women is to be formed from a group of 9 men and 7 women. In how many different ways can the committee be formed? **1**

(b) PQ and SR are two towers of height 0.7 km. Q is 10 km due North of R.
A vehicle at T is travelling due East moving away from Q at a constant speed of 10 km/h.
Let the distance QT be x, the distance PT be k, and $\angle PTS$ be θ.



(i) Show that: **2**

$$\frac{dk}{dt} = \frac{10x}{\sqrt{x^2+0.49}}.$$

(ii) By first finding an expression for k in terms of θ, show that **2**

$$\frac{dk}{d\theta} = -10\ \mathrm{cosec}^2\theta.$$

(iii) Find the exact rate at which θ is changing when the vehicle is 2.4 km from Q. Leave your answer correct to 2 decimal places. **3**

(c) Find the exact value of $\cos^{-1}(\sin\frac{4\pi}{3})$. **2**

(d) Use the substitution $u = \tan x - x$ to find $\int \tan^2 x\sqrt{\tan x - x}\, dx$. **2**

Section II continued

Question 15 (11 marks) Use a SEPARATE writing booklet. **Marks**

(a) At a film festival, there are nine feature films. On the opening night, all 9 films are shown at 6pm, 8pm as well as 10pm. **2**

There are 200 film review critics at the opening night of the festival.

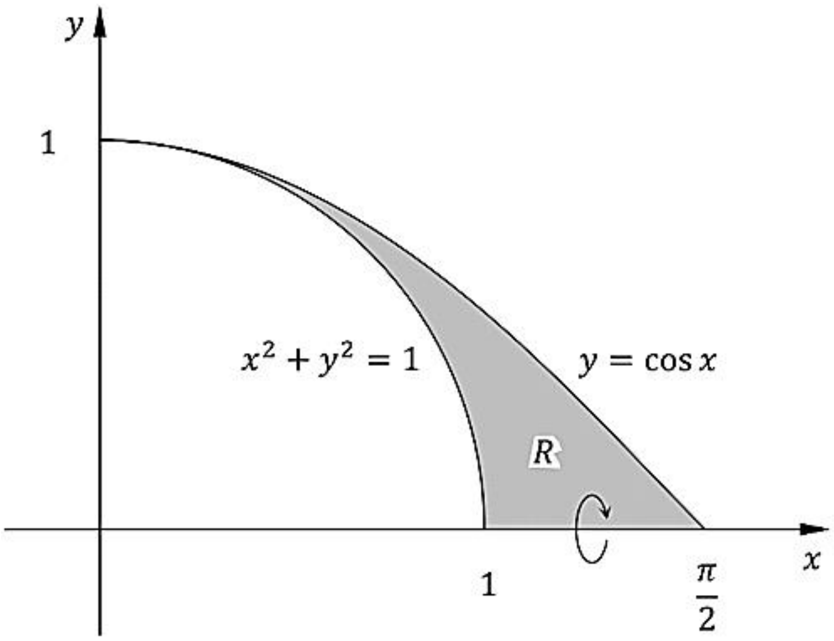
Each critic attends one movie at 6pm, a different movie at 8pm and then a third different movie at 10 pm. Each critic will end up seeing 3 different films in total.

Use the pigeonhole principle to show that at least one combination of three films will be reviewed by at least three different critics.

(b) (i) Differentiate $x\tan^{-1}x$ with respect to x. **1**

(ii) Hence, find $\int \tan^{-1}x \; dx$. **2**

(c) The region R is bounded by the x-axis, the y-axis, $y = \cos x$ and the unit circle, as shown in the diagram. **3**
The graph of $y = \cos x$ lies on or outside the unit circle.



Find the exact volume of the solid of revolution formed when the region R is rotated about the x-axis.

(d) The Eiffel tower in Paris of height h metres stands with its base at a point O on a horizontal ground. At the same instant, stone A is projected from O with speed V ms^{-1} at an angle α above the horizontal and stone B is projected from the top of the tower with speed U ms^{-1} at an angle β above the horizontal, where $\alpha > \beta$. **3**

The two stones move in the same vertical plane under gravity, where the acceleration due to gravity is g ms^{-1}, and collide after T seconds. At time t seconds after firing, the position vectors of the two stones are:

$\underset{\sim}{r}_A(t) = (Vt\cos\alpha)\underset{\sim}{i} + \left(-\frac{1}{2}gt^2 + Vt\sin\alpha\right)\underset{\sim}{j}$ and

$\underset{\sim}{r}_B(t) = (Ut\cos\beta)\underset{\sim}{i} + \left(h - \frac{1}{2}gt^2 + Ut\sin\beta\right)\underset{\sim}{j}.$

DO NOT PROVE THE ABOVE RESULTS.

Show that $T = \dfrac{h\cos\alpha}{U\sin(\alpha-\beta)}$.

END OF EXAMINATION

Question Number	Suggested Solutions	Marking Guidelines/ Teachers' Comments
1.	Range: $0 \leq \frac{y}{4} \leq \pi \;\therefore 0 \leq y \leq 4\pi$.	1 Mark: C
2.	Let α, β, γ be the roots of $P(x) = 3x^3 + 2x^2 - x + 5 = 0$. $\alpha\beta\gamma = -\frac{d}{a} = -\frac{5}{3}$	1 Mark: D
3.	As $h(x)$ has 4 zeros, the graphs of $f(x)$ and $g(x)$ must have a total of 4 distinct zeros, which means the correct option must be B or D. Option B has 2 of its zeros on the negative part of the x axis, but $h(x)$ has only one zero on the negative part of the x axis, another zero at the origin and 2 zeros on the positive part of the x axis, which matches with the zeros of the 2 graphs in option D exactly. Hence, the correct option is **D**.	1 Mark: D
4.	$\cos^2 nx = \frac{1}{2}(1 + \cos nx)$ $\int \cos^2 7x \; dx = \frac{1}{2} \int (1 + \cos 14x) \; dx$ $= \frac{1}{2}\left(x + \frac{1}{14} \sin 14x\right) + C$ $= \frac{x}{2} + \frac{1}{28} \sin 14x + C$	1 Mark: B
5.	$\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4}{\sqrt{9-x^2}} dx = 4 \times \int_{\frac{3}{\sqrt{2}}}^{3} \frac{1}{\sqrt{9-x^2}} dx$ $= 4\left[\sin^{-1}\left(\frac{x}{3}\right)\right]_{\frac{3}{\sqrt{2}}}^{3}$ $= 4\left[\sin^{-1}\left(\frac{3}{3}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$ $= 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$ $= \pi$	1 Mark: D
6.	$\frac{10}{8} = 1.25$ $\therefore 1 + e^{-0.12(t-20)} = 1.25$ $e^{-0.12(t-20)} = 0.25$ $-0.12(t-20) = \ln 0.25$ $\therefore t = 31.6$	1 Mark: A

7.

$(a+b)^n$

$= \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots$

$+ \binom{n}{n} a^0 b^n$

General term $= \binom{20}{r} x^{60-3r} \left(\frac{\mathbf{2}}{\boldsymbol{x}}\right)^r$

$= \binom{20}{r} 2^r x^{60-4r}$

For the term independent of x,

$60 - 4r = 0 \Rightarrow r = 15$

$\therefore$ The independent term $= \binom{20}{15} 2^{15}$

$= \binom{20}{5} 2^{15}$

1 Mark: B

8.

We have $-1 \le t \le 2$ this means
$-2 \le t-1 \le 1$ and as from the graph
$-2 \le x \le 1$ we can easily deduce that $x = t-1$.
This indicates that options B and C are invalid.
Also, using the following table below we can see that option D is invalid as $(1, 14)$ is not a point on the graph.

$x = t-1, y = 3t + t^3$

	t	x	y
Option D	2	1	14

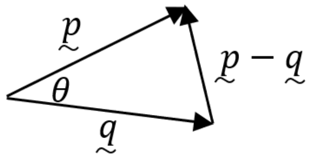
Now, Options B, C and D are invalid then option A must be valid. In addition, if we substitute any value of t such that $-1 \le t \le 2$ we obtain a point on the curve.
Hence, the correct option is **A.**

$x = t - 1$ and $y = 3t - t^3$

	t	x	y
Option A	2	1	-2

1 Mark: A

9.



Using the cosine rule

$|\underset{\sim}{p} - \underset{\sim}{q}|^2 = |\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - 2|\underset{\sim}{p}||\underset{\sim}{q}| \cos\theta$

When $\theta = \frac{\pi}{3}$, $|\underset{\sim}{p} - \underset{\sim}{q}|^2 = 9 + 4 - 2 \times 3 \times 2 \times \frac{1}{2}$

$|\underset{\sim}{p} - \underset{\sim}{q}| = \sqrt{13-6} = \sqrt{7}$

When $\theta = \frac{2\pi}{3}$, $|\underset{\sim}{p} - \underset{\sim}{q}|^2 = 9 + 4 - 2 \times 3 \times 2 \times \left(-\frac{1}{2}\right)$

$|\underset{\sim}{p} - \underset{\sim}{q}| = \sqrt{13+6} = \sqrt{19}$

So, $\sqrt{7} \le |\underset{\sim}{p} - \underset{\sim}{q}| \le \sqrt{19}$

Hence, the correct option is **C.**

1 Mark: C

10.	The length of the vector projection of vector $\underset{\sim}{p}$ onto vector $\underset{\sim}{q}$ is $\frac{\underset{\sim}{p}\cdot \underset{\sim}{q}}{\lvert \underset{\sim}{q} \rvert}$ which we are given as $\frac{32}{13}$. Now, $\lvert \underset{\sim}{q} \rvert = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$ and $\underset{\sim}{p} \cdot \underset{\sim}{q} = -10\,(\ln x)^2 + 36 \ln x$ this indicates that $\frac{\underset{\sim}{p}\cdot \underset{\sim}{g}}{\lvert \underset{\sim}{q} \rvert} = \frac{-10\,(\ln x)^2 + 36 \ln x}{13} = \frac{32}{13}$ that is $-10\,(\ln x)^2 + 36 \ln x = 32$ $10\,(\ln \mathrm{x})^2 - 36 \ln x = -32$ $10\,(\ln x)^2 - 36 \ln x + 32 = 0$ $Let\ m = lnx$ $10\,m^2 - 36\,m + 32 = 0$ $2\,(5m - 8)(m - 2) = 0$, that is, $m = 2\ or\ m = \frac{8}{5}$. As $m = \ln x$ is a positive integer then only $\ln x = 2$ is valid. $$\underset{\sim}{p} = 2\,(\ln x)^2\, \underset{\sim}{i} + 3 \ln x\, \underset{\sim}{j}$$ Therefore, $p = 8\,\underset{\sim}{i} + 6\,\underset{\sim}{j}$ so $\lvert p \rvert = \sqrt{8^2 + 6^2} = 10$. Hence, the correct option is **B**.	1 Mark: B

Question 11

EXAMINER'S COMMENTS

(a) (i) $(x^2+y)^4$

$$= \binom{4}{0}(x^2)^4 y^0 + \binom{4}{1}(x^2)^3 y^1 + \binom{4}{2}(x^2)^2 y^2 + \binom{4}{3}(x^2)^1 y^3 + \binom{4}{4}(x^2)^0 y^4 \quad (1)$$

$$= x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$$

(Using the Binomial Theorem)

- The students were not penalised if the highlighted parts were missing (but they should learn to write it out in full).

- Students lost half a mark if $\binom{4}{r}$ was not used or $(x^2)^r y^{4-r}$ was not used.

- Some students wrote out Pascal's triangle, and this was accepted to obtain the coefficients.

Alternative Solution (not recommended)

$$(x^2+y)^2(x^2+y)^2 = (x^4+2x^2y+y^2)(x^4+2x^2y+y^2)$$

$$= x^8 + 2x^6y + x^4y^2 + 2x^6y + 4x^4y^2 + 2x^2y^3 + x^4y^2 + 2x^2y^3 + y^4$$

$$= x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$$

Question 11

EXAMINER'S COMMENTS

(a)(ii) In previous expansion, let

$x^2 = b^4$ and $y = -1$

$\therefore x = b^2$ and $y = -1$ 1

$\therefore (b^4 - 1)^2 =$

$(b^2)^8 + 4(b^2)^6(-1) + 6(b^2)^4(-1)^2 + 4(b^2)^2(-1)^3 + (-1)^4$

$= b^{16} - 4b^{12} + 6b^8 - 4b^4 + 1$ 1

(b) Method 1 – multiply both sides by $(x-3)^2$.

$(x-3)^2 \times \dfrac{1}{x-3} \geq -1 \times (x-3)^2$ 1

$x - 3 \geq -(x^2 - 6x + 9)$

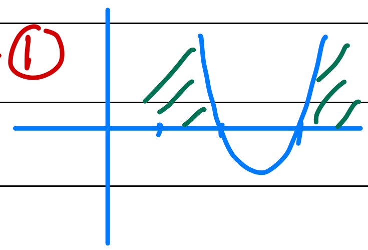
$x - 3 \geq -x^2 + 6x - 9$

$x^2 - 5x + 6 \geq 0$

$(x-2)(x-3) \geq 0$ 1

$x \leq 2$ or $x > 3$ 1

as $x \neq 3$.



- Lose $\frac{1}{2}$ if $x = 3$ included in the answer.

- CFPE allowed for last step.

Question 11

EXAMINER'S COMMENTS

(b) Method 2 – Critical Point Method

$$\frac{1}{x-3} \geqslant -1$$

$$\frac{1}{x-3} + 1 \geqslant 0$$

$$\frac{1+x-3}{x-3} \geqslant 0 \quad \text{———} \ 1$$

zero at $x = 2$

$$\frac{x-2}{x-3} \geqslant 0$$

discontinuity at $x = 3$

x	1	2	2.5	3	4
$x-2$	-1	0	0.5	1	2
$x-3$	-2	-1	-0.5	0	1
$\frac{x-2}{x-3}$	$+\frac{1}{2}$	0	-1	Un-defined	$+2$

——— 1

From the table we can see that $\frac{x-2}{x-3} \geqslant 0$ when

$x \leq 2$ or $x > 3$. ——— 1

Question 11

EXAMINER'S COMMENTS

(c)(i) $P(x) = 2x^3 + mx^2 + nx - 20$

$P(2) = 0$

$2(2)^3 + m(2)^2 + n(2) - 20 = 0$

$16 + 4m + 2n - 20 = 0$

$4m + 2n = 4$ —— 1

$P'(x) = 6x^2 + 2mx + n$

$P'(2) = 0$

$6(2)^2 + 2m(2) + n = 0$

$24 + 4m + n = 0$

$4m + n = -24$ —— 1

$4m + 2n = 4$ — (1)

$4m + n = -24$ — (2)

(1) − (2) : $n = 4 - (-24) = 28$

SUB $n = 28$ into (2) : $4m + 28 = -24$

$4m = -52$

$m = -13$

$\therefore m = -13, \; n = 28$ —— 1

EXAMINER'S COMMENTS

(c) (ii) Using $m = -13$, $n = 28$:

$$P(x) = 2x^3 + mx^2 + nx - 20$$

$$P(x) = 2x^3 - 13x^2 + 28x - 20$$

From $P(2) = P'(2) = 0$, we know that $x = 2$ is a double root.

$\therefore$ The roots are 2, 2, and α.

Using coefficients of the Polynomial, the sum of roots equals $-\frac{b}{a}$.

$\therefore 2 + 2 + \alpha = \frac{-(-13)}{2}$ ——— 1

$$4 + \alpha = \frac{13}{2}$$

$$\alpha = \frac{13}{2} - \frac{8}{2}$$

$\therefore$ Other root: $\alpha = \frac{5}{2}$ ——— 1

- Students could also use product of roots,
$2 \times 2 \times \alpha = -\frac{(-20)}{2}$

- Students could also divide $P(x)$ by $(x-2)^2 = x^2 - 4x + 4$. A number of students only got 1 out of 2, as they factorised but didn't solve and find root.

Question 11

EXAMINER'S COMMENTS

(d) $9\sin x + 40\cos x = R\sin(x+\alpha)$

$9\sin x + 40\cos x = R[\sin x\cos\alpha + \cos x\sin\alpha]$

$= R\cos\alpha\sin x + R\sin\alpha\cos x$

$\therefore$ Equating coefficients :

$R\cos\alpha = 9$

$R\sin\alpha = 40$

$R^2\cos^2\alpha + R^2\sin^2\alpha = 9^2 + 40^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 1681$

$R^2 = 1681$

$R = \sqrt{1681} = 41$ —— $\frac{1}{2}$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{40}{9}$

$\tan\alpha = \frac{40}{9}$ —— $\frac{1}{2}$

$\alpha = \tan^{-1}\left(\frac{40}{9}\right) = 1.35$ —— $\frac{1}{2}$

- Has to be in radians and to 3 significant figures.

$\therefore 9\sin x + 40\cos x = 41\sin(x + 1.35)$ —— $\frac{1}{2}$

MATHEMATICS EXTENSION 1 – QUESTION 12 HSC Trial Exam 2023

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) As $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{a}$, then		
$\underset{\sim}{a} \cdot \underset{\sim}{b} = x - 2y = 0$, that is $x = 2y$ --- (1)	1mk	
Also, $\lvert \underset{\sim}{b} \rvert = \sqrt{x^2 + y^2} = 2\sqrt{5}$	1mk	
By squaring both sides, we get		
$x^2 + y^2 = 20$ --- (2)		
By substituting (1) into (2), we get		
$(2y)^2 + y^2 = 20$		
$4y^2 + y^2 = 20$		
$5y^2 = 20$		
$y^2 = 4$		
$y = \pm 2$		
$x = 2y$ $\quad$ $x = 2y$		
$= 2(2)$ $\quad$ $= 2(-2)$		
$= 4$ $\quad$ $= -4$		
$\underbrace{x = 2,\ y = 4}_{\frac{1}{2}\text{mk}}$ $\quad$ $\underbrace{x = 2,\ y = -4}_{\frac{1}{2}\text{mk}}$		
This question was generally well done. However, a few students did not consider both the positive and negative square roots of y thus ending up with only one set of x and y-values.		

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) $\frac{d}{dx}\left(\cos^{-1} f(x)\right) = \frac{-f'(x)}{\sqrt{1-(f(x))^2}}$		
$\therefore \frac{d}{dx}\left(\cos^{-1} 2x\right) = \frac{-2}{\sqrt{1-(2x)^2}}$		
$= \frac{-2}{\sqrt{1-4x^2}}$	1 mk	
Therefore, the gradient of tangent at $x=0$ is: $\frac{-2}{\sqrt{1-4(0)^2}} = \frac{-2}{1} = -2$	½ mk	
The point is:		
$x=0$		
$y = \cos^{-1}(2(0))$		
$= \pi/2$	½ mk	
Equation of tangent at $(0, \pi/2)$ is:		
$y - y_1 = m(x - x_1)$		
$y - \pi/2 = -2(x-0)$		
Therefore, the equation of the tangent is:		
$y - \pi/2 = -2x$		
$\therefore y = -2x + \pi/2$	1 mk	
This question was generally well done. Some students did not realise that $\cos^{-1}(0) = \pi/2$. (instead wrote down $\cos^{-1}(0) = 0$ which is incorrect).		

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) Let $P(n)$ represent the proposition.		
Base case – Prove true for $n=1$		
$P(1)$ is true since		
$LHS = 2^1 + 2 = 2 + 2 = 4$ $RHS = 2^{1+1} + 1(1+1) - 2 = 2^2 + 2 - 2 = 2^2 = 4$		
$\therefore$ LHS = RHS		
$\therefore$ proved true for $n=1$.	1mk	
Inductive Hypothesis:		
Assume the statement is true for $n=k$.		
If $P(k)$ is true for some arbitrary $k \geq 1$, then		
$(2^1+2) + (2^2+4) + (2^3+6) + \dots + (2^k + 2k) = 2^{k+1} + k(k+1) - 2$		
Prove that the statement is true for $n=k+1$		
Required to Prove $P(k+1)$:		
$(2^1+2) + (2^2+4) + (2^3+6) + \dots + (2^k+2k) + 2^{k+1} + 2(k+1) = 2^{k+2} + (k+1)(k+2) - 2$		
$LHS = \underbrace{2^{k+1} + k(k+1) - 2}_{\text{from inductive hypothesis}} + 2^{k+1} + 2(k+1)$		1mk for using the inductive step appropriately.
$= 2^1 \cdot 2^{k+1} + k(k+1) + 2(k+1) - 2$		1mk – for appropriate algebraic manipulation
$= 2^{k+2} + (k+1)(k+2) - 2 = RHS$		
important to display this step.		
$\therefore$ Proved the statement is true for $n=k+1$		
Hence, $P(n)$ is true for $n \geq 1$ by the principle of mathematical induction		
Generally, well attempted. However, the highlighted concept was poorly attempted / displayed.		

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(d) From the reference sheet		
$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$		
So, $\cos 4x \cos 2x = \frac{1}{2}\left[\cos(4x-2x) + \cos(4x+2x)\right]$		
$\cos 4x \cos 2x = \frac{1}{2}\left[\cos 2x + \cos 6x\right]$		
$\int_0^{\pi/3} (\cos 4x \cdot \cos 2x)\, dx$		
$= \frac{1}{2}\int_0^{\pi/3} \left[\cos 2x + \cos 6x\right] dx$	1mk	
$= \frac{1}{2}\left[\frac{1}{2}\sin 2x + \frac{1}{6}\sin 6x\right]_0^{\pi/3}$	1mk	
$= \frac{1}{2}\left[\frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{6}\sin 2\pi - 0\right]$		
$= \frac{1}{2}\left[\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{6} \times 0 - 0\right]$		
$= \frac{\sqrt{3}}{8}$	1mk	
Quite well done. A few students attempted longer and more complex methods such as using the double angle followed by integration using u- substitution.		

MATHEMATICS EXTENSION 1 – QUESTION 13 2023 Trial HSC		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\underset{\sim}{r}(t) = \begin{pmatrix} Vt\cos\alpha \\ -\frac{1}{2}gt^2 + Vt\sin\alpha \end{pmatrix}$		
$\therefore x = Vt\cos\alpha$ ---(1)		
$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$ ---(2)		
If $\alpha = 28$ and $x_R = 60$		
sub. in (1)		
$x = Vt\cos\alpha$		
$60 = Vt\cos 28$		
$t = \dfrac{60}{V\cos 28}$	1	
when $y = 0$, $t = \dfrac{60}{V\cos 28}$		
sub in (2)		
$0 = -\frac{1}{2}g\left(\dfrac{60}{V\cos 28}\right)^2 + V\left(\dfrac{60}{V\cos 28}\right)\sin 28$	1	
$\frac{1}{2}g\left(\dfrac{3600}{V^2\cos^2 28}\right) = 60\tan 28$		
$\dfrac{3600}{V^2\cos^2 28} = \dfrac{120\tan 28}{g}$		
$V^2\cos^2 28 = \dfrac{3600g}{120\tan 28}$		
$V^2 = \dfrac{3600g}{120\tan 28} \div \cos^2 28$		
$\doteqdot 709.256\ldots$		
$V \doteqdot 26.6$ m/s $V > 0$	1	
		3

MATHEMATICS EXTENSION 1 – QUESTION 13 2023 Trial HSC		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) $4x^3 - 3x + 1 = 0$ ---- (1)		
Let $x = \cos\theta$ sub in (1)		
$\therefore 4\cos^3\theta - 3\cos\theta + 1 = 0$		
$\cos 3\theta + 1 = 0$		
$\cos 3\theta = -1$	1	
$\therefore 3\theta = \pi, 3\pi, 5\pi, \ldots$		
$\therefore \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}, \ldots$	1	for 3 values of θ ($\frac{1}{2}$ mark for 2 values of θ)
$\therefore x = \cos\frac{\pi}{3}, \cos\pi, \cos\frac{5\pi}{3}$		
but $\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$ 		
$\therefore x = \frac{1}{2}, -1, \frac{1}{2}$		$\frac{1}{2}$ mk for showing $x = \frac{1}{2}$ is a repeated root
$\therefore$ There are only 2 distinct solutions to this cubic equation, $x = \frac{1}{2}$ (double root) and $x = -1$		$\frac{1}{2}$ mk for giving the other root $x = -1$ (or $x = \cos\pi$)

MATHEMATICS EXTENSION 1 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) $(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = \underset{\sim}{a}\cdot\underset{\sim}{a} - \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{a} - \underset{\sim}{b}\cdot\underset{\sim}{b}$	1	For correct expansion
$= \underset{\sim}{a}\cdot\underset{\sim}{a} - \cancel{\underset{\sim}{a}\cdot\underset{\sim}{b}} + \cancel{\underset{\sim}{a}\cdot\underset{\sim}{b}} - \underset{\sim}{b}\cdot\underset{\sim}{b}$		Many students skipped this step
$= \underset{\sim}{a}\cdot\underset{\sim}{a} - \underset{\sim}{b}\cdot\underset{\sim}{b}$	$\frac{1}{2}$	
but $\underset{\sim}{a}\cdot\underset{\sim}{a} = \lvert\underset{\sim}{a}\rvert^2$	$\frac{1}{2}$ mk	For stating this property
$\therefore (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = \lvert\underset{\sim}{a}\rvert^2 - \lvert\underset{\sim}{b}\rvert^2$		
ii) If AOBC is a rhombus the the adjacent sides are equal in length ie $\lvert\underset{\sim}{a}\rvert = \lvert\underset{\sim}{b}\rvert$	$\frac{1}{2}$	
So $(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = \lvert\underset{\sim}{a}\rvert^2 - \lvert\underset{\sim}{b}\rvert^2$ from (i)		
$= \lvert\underset{\sim}{a}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2$		This line was not used by many students
$= 0$		
Since the dot product of the 2 vectors is zero then they are perpendicular $\therefore$ the diagonals are perpendicular	$\frac{1}{2}$	
NB// Many students did not use the second line in their working. Even though full marks were awarded, it is important to include <u>ALL</u> steps in a SHOW question.		

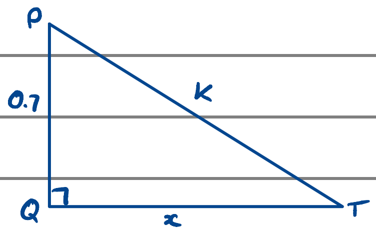
MATHEMATICS EXTENSION 1 – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) There are $10! = 3\,628\,800$ ways that 10 tracks can be arranged in a play list.	1	
The 2 D'Arcy tracks are played together are one group and the 3 G Flip tracks played together are one group. The other 5 tracks are separately in their own group as 1 track.		
$\underbrace{\boxed{2}}_{1}$ + $\underbrace{\boxed{3}}_{1}$ + $\underbrace{\_\ \_\ \_\ \_\ \_}_{5}$ D'Arcy G Flip		
So as we have 7 groups of tracks then they can be arranged in $7!$ ways. Within each arrangement, the 2 tracks by D'Arcy can be arranged in $2!$ ways & the 3 tracks by G Flip can be arranged in $3!$ ways	1	
$\therefore$ the number of shuffle orders that the songs can be played in is: $7! \times 3! \times 2! = 60\,480$ ways		
$\therefore$ probability $= \dfrac{7! \times 3! \times 2!}{10!}$ $= \dfrac{1}{60}$		

MATHEMATICS EXTENSION 1 – QUESTION 13		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
3 marks: Correct answer, with working.		
2 marks: - Finds $10! = 3\,628\,800$		
AND		
- Uses $3!$ and $2!$ in the right context.		
1 mark: - Finds $10! = 3\,628\,800$		
OR		
- Uses $3!$ and $2!$ in an attempt to find the number of ways 3 G Flip and 2 D'Arcy tracks can be arranged.		

a) $^9C_4 \times {}^7C_2 = 2646$

b) i In ΔPQT,



$k^2 = x^2 + 0.7^2$ (Pythagoras' Theorem)

$k = \sqrt{x^2 + 0.49}$ ($k > 0$, length)

$= (x^2 + 0.49)^{\frac{1}{2}}$

$\therefore \frac{dk}{dx} = \frac{1}{2} \times 2x \times (x^2 + 0.49)^{-\frac{1}{2}}$

$= \frac{x}{\sqrt{x^2 + 0.49}}$

Also $\frac{dx}{dt} = 10$, since the vehicle is moving at 10km/h

$\frac{dk}{dt} = \frac{dk}{dx} \times \frac{dx}{dt}$

$= \frac{x}{\sqrt{x^2 + 0.49}} \times 10$

$= \frac{10x}{\sqrt{x^2 + 0.49}}$

2 marks Complete solution, including showing where $k = \sqrt{x^2 + 0.49}$ comes from.

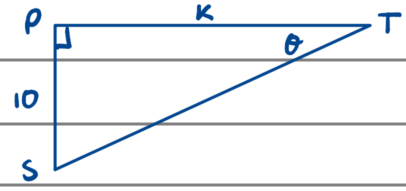
1 mark Clearly shows the relationship $k^2 = x^2 + 0.7^2$

or Correctly combines $\frac{dk}{dx}$ and $\frac{dx}{dt}$, without showing where $k = \sqrt{x^2 + 0.49}$ comes from.

This question was poorly done. This is a "show" question; don't leave **anything** out. Feedback from HSC examiners is to show even more detail in a "show" or "prove" question, to convince the marker you know exactly where every part of the result comes from, and you haven't just worked backwards from what you're given. In this case, the expression $\sqrt{x^2 + 0.49}$ is given in the question, so it is important to show where this comes from. Quoting it as the start of your working is not enough.

MATHEMATICS EXTENSION 1 – QUESTION 14 (continued)

b) ii In ΔPTS,



$\tan\theta = \frac{10}{k}$

$\therefore k = \frac{10}{\tan\theta}$

$= 10\tan\theta^{-1}$

$\frac{dk}{d\theta} = -1 \times 10 \times \sec^2\theta \times \tan\theta^{-2}$

$= -10 \times \frac{1}{\cos^2\theta} \times \left(\frac{\sin\theta}{\cos\theta}\right)^{-2}$

$= -10 \times \frac{1}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$

$= -10 \times \frac{1}{\sin^2\theta}$

$= -10\,\mathrm{cosec}^2\theta$

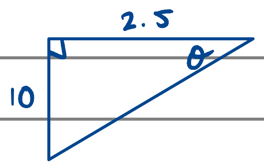
2 marks Complete solution

1 mark Correctly finds an expression for k in terms of θ (ie $k = \frac{10}{\tan\theta}$)

Again, this question was poorly done (see previous comment). It is certainly true that the derivative of $(\tan\theta)^{-1}$ is $-\mathrm{cosec}^2\theta$, but it's your job to **show** it! Don't skip steps, or do them in your head; write everything down.

iii When $x = 2.4$, $k = \sqrt{2.4^2 + 0.49}$ (from i)

$= 2.5$

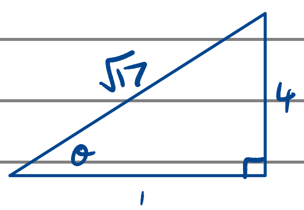


when $k = 2.5$,

$\tan\theta = \frac{10}{2.5}$

$= 4$

∴



$\therefore \sin\theta = \frac{4}{\sqrt{17}}$

$\therefore \mathrm{cosec}\,\theta = \frac{\sqrt{17}}{4}$

b) iii (continued)

$$\frac{d\theta}{dt} = \frac{d\theta}{dk} \times \frac{dk}{db}$$

$$= \frac{1}{-10\operatorname{cosec}^2\theta} \times \frac{10x}{\sqrt{x^2+0.49}}$$

$$= \frac{1}{-10\left[\frac{\sqrt{17}}{4}\right]^2} \times \frac{10(2.4)}{\sqrt{2.4+0.49}}$$

$$= \frac{-16}{17} \times \frac{24}{25}$$

$$= \frac{-384}{425}$$

$$= -0.90352\ldots$$

$$\doteqdot -0.90 \text{ radian per hour}$$

3 marks Complete solution

2 marks Correctly finds $k=2.5$ and $\operatorname{cosec}\theta = \frac{\sqrt{17}}{4}$

1 mark Correctly finds $k=2.5$

or correctly combines rates to get an expression for $\frac{d\theta}{dt}$ that depends only on θ.

Many students had difficulty with this question. Many also did not show appropriate working to earn partial marks. If your work is not set out clearly you risk earning nothing (or at best 1 mark) from this 3-mark question.

Also note that calculus with trigonometry only works for angles in radians.

c) let $\alpha = \cos^{-1}\left(\sin \frac{4\pi}{3}\right)$

$\therefore \cos\alpha = \sin\frac{4\pi}{3}$, where $0 \le \alpha \le \pi$

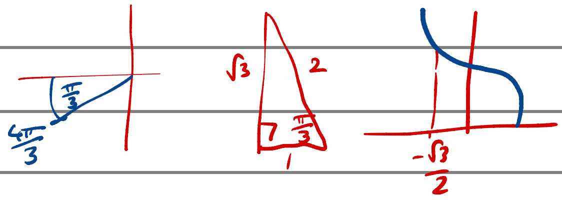
$= \frac{-\sqrt{3}}{2}$

$\therefore$ related angle $\frac{\pi}{6}$, 2nd quadrant

$\therefore \alpha = \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

$\therefore \cos^{-1}\left(\sin\frac{4\pi}{3}\right) = \frac{5\pi}{6}$



2 marks — Complete solution

1 mark — Correctly identifying $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

Note that your calculator can tell you that $\cos^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = 2.61799\ldots$
Dividing this number by pi gives you $\frac{5}{6}$, which is quite useful.

d) $\int \tan^2 x\sqrt{\tan x - x}\, dx$

let $u = \tan x - x$

$\frac{du}{dx} = \sec^2 x - 1$

$= \tan^2 x$ (since $1 + \tan^2 x = \sec^2 x$)

$du = \tan^2 x\, dx$

$= \int \sqrt{\tan x - x}\, \tan^2 x\, dx$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}} du$

$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3}(\tan x - x)^{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{(\tan x - x)^3} + C$

Mostly well done. In this case marks were not deducted for mixing variables, but they may be in future. Your integral should be entirely in x, or entirely in u, but never a mix of the two.

2 marks — Complete solution

$1\frac{1}{2}$ marks — Complete solution, without "+c"

1 mark — Correctly reducing the integral to $\int \sqrt{u}\, du$

MATHEMATICS EXTENSION 1 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Need to choose 3 movies from the 9 $^9C_3 = 84$. $\frac{200}{84} = 2.38095238$ $\quad \left\lceil \frac{200}{84} \right\rceil = 3$	1	Correctly finding $^9C_3 = 84$ ½ mark lost for Incorrect notation when ceiling function was used.
This means each of 84 critics could (at worst case) choose a different combination of three movies. The next 84 critics could also choose these same combinations of movies (as we have already had critics view each of the combinations) (168 critics accounted for) Now the next critic would need to watch a combination of three movies that has already been watched by two other critics.	1	Clearly showing that at least one combination of three films will be reviewed by at least three different critics.
∴ By the pigeonhole principle at least one combination of three films will be reviewed by at least three different critics.		

- Learn the correct ceiling function notation.
- Be clear with explanation
- Avoid using ≑ or ≈.

MATHEMATICS EXTENSION 1 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) (i) Let $y = x\tan^{-1}x$ $\quad u = x \quad v = \tan^{-1}x$		
$y = uv \quad u' = 1 \quad v' = \frac{1}{1+x^2}$		
$y' = vu' + uv'$		
$= \tan^{-1}x + \frac{x}{1+x^2}$	1	Correctly differentiates $y = x\tan^{-1}x$
(ii) METHOD 1		
$\int \tan^{-1}x \, dx$		
$= \int \left(\tan^{-1}x + \frac{x}{1+x^2} - \frac{x}{1+x^2}\right) dx$	1	Correctly sets up integral.
$= \int \left(\tan^{-1}x + \frac{x}{1+x^2}\right) dx - \frac{1}{2}\int \frac{2x}{1+x^2} dx$		
$= x\tan^{-1}x - \frac{1}{2}\ln\|1+x^2\| + C$	1	Correct answer
METHOD 2		
$\frac{d}{dx}(x\tan^{-1}x) = \tan^{-1}x + \frac{x}{1+x^2}$		
$\therefore \int \frac{d}{dx}(x\tan^{-1}x) dx = \int \left(\tan^{-1}x + \frac{x}{1+x^2}\right) dx$		
$x\tan^{-1}x = \int \tan^{-1}x \, dx + \frac{1}{2}\int \frac{2x}{1+x^2} dx$		
$x\tan^{-1}x = \int \tan^{-1}x \, dx + \frac{1}{2}\ln\|1+x^2\| + C_1$		
$\therefore \int \tan^{-1}x \, dx = x\tan^{-1}x - \frac{1}{2}\ln\|1+x^2\| + C_2$		

MATHEMATICS EXTENSION 1 – QUESTION 15

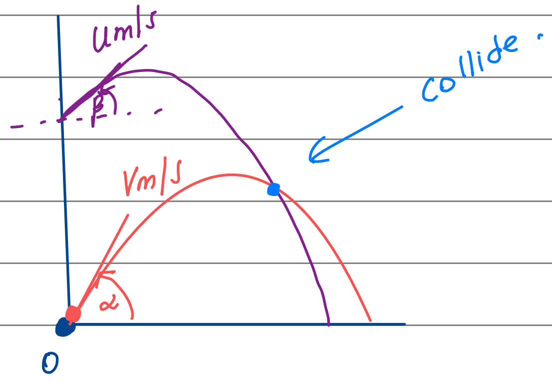
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $V = \pi \int_0^{\pi/2} \overset{V_1}{\cos^2 x} \, dx - \overset{V_2}{\frac{1}{2} \times \frac{4}{3}\pi r^3}$	1	Correct integration with correct bounds to find V_1
$V = \frac{\pi}{2}\int_0^{\pi/2} (\cos 2x + 1)\, dx - \frac{2}{3}\pi(1)^3$		
$= \frac{\pi}{2}\left[\frac{1}{2}\sin 2x + x\right]_0^{\pi/2} - \frac{2}{3}\pi$	1	Correctly found Volume of the hemisphere V_2
$= \frac{\pi}{2}\left[\frac{1}{2}\sin\pi + \frac{\pi}{2} - (\sin 0 + 0)\right] - \frac{2\pi}{3}$		
$= \frac{\pi}{2}\left(0 + \frac{\pi}{2} - 0\right) - \frac{2\pi}{3}$	1	Correct answer.
$= \frac{\pi^2}{4} - \frac{2\pi}{3}$ units^3		
$= \frac{3\pi^2 - 8\pi}{12}\ u^3$		If an incorrect process was used to gain 1/3 the student needed to square π, apply the identity AND then integrate it correctly.
$= \frac{\pi(\pi - 8)}{12}\ u^3$		
METHOD 2		
$V = \pi\int_0^{\pi/2} \overset{V_1}{\cos^2 x \, dx} - \overset{V_2}{\pi\int_0^1 (1 - x^2)\, dx}$		
$V = \frac{\pi}{2}\int_0^{\pi/2}(\cos 2x + 1)\, dx - \pi\left[x - \frac{x^3}{3}\right]_0^1$	1	Correct integration with correct bounds to find the volume V_1
$= \frac{\pi}{2}\left[2\sin 2x + x\right]_0^{\pi/2} - \pi\left[1 - \frac{1}{3}\right]$		
$= \frac{\pi}{2}\left(2\sin\pi + \frac{\pi}{2} - 0\right) - \frac{2\pi}{3}$	1	Correct integration with correct bounds to find V_2
$= \frac{\pi^2}{4} - \frac{2\pi}{3}$	1	Correct answer
The question was poorly executed by many students. Most common mistake was: $\int_0^{\pi/2}\left[\cos^2 x - (x^2 - 1)\right] dx$		

MATHEMATICS EXTENSION 1 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
METHOD 3		
$V_1 = \pi\int_0^1 \left[\cos^2 x - (1-x^2)\right] dx$		
$= \pi\int_0^1 \left(\frac{1}{2} + \frac{1}{2}\cos 2x - 1 + x^2\right) dx$		
$= \pi\int_0^1 \left(\frac{1}{2}\cos 2x - \frac{1}{2} + x^2\right) dx$		
$= \pi\left[\frac{1}{4}\sin 2x - \frac{1}{2}x + \frac{x^3}{3}\right]_0^1$		
$= \pi\left(\frac{1}{4}\sin 2 - \frac{1}{2} + \frac{1}{3}\right) = \pi\left[\frac{1}{4}\sin 2 - \left(\frac{1}{2} - \frac{1}{3}\right)\right]$		
$= \frac{\pi}{4}\sin 2 - \frac{\pi}{6}$		
$V_2 = \pi\int_1^{\pi/2} \cos^2 x \, dx$		
$= \frac{\pi}{2}\int_1^{\pi/2}(1 + \cos 2x)\, dx$		
$= \frac{\pi}{2}\left[x + 2\sin 2x\right]_1^{\pi/2}$		
$= \frac{\pi}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin\pi - 1 - \frac{1}{2}\sin 2\right]$		
$= \frac{\pi}{2}\left(\frac{\pi}{2} - 1 - \frac{1}{2}\sin 2\right)$		
$= \frac{\pi^2}{4} - \frac{\pi}{2} - \frac{\pi}{4}\sin 2$		
$\therefore V = V_1 + V_2$		
$= \frac{\pi}{4}\sin 2 - \frac{\pi}{6} + \frac{\pi^2}{4} - \frac{\pi}{2} - \frac{\pi}{4}\sin 2$		
$= \frac{\pi^2}{4} - \frac{8\pi}{12}$		

$= \frac{\pi^2}{4} - \frac{2\pi}{3}$

MATHEMATICS EXTENSION 1 – QUESTION 15



SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
METHOD 2 At $t = T$ they collide		
$VT\cos\alpha = UT\cos\beta$ $\quad \cancel{-\frac{1}{2}gT^2} + VT\sin\alpha = h \cancel{-\frac{1}{2}gT^2} + UT\sin\beta$	1	Equate x and y components.
$\therefore \frac{V\cos\alpha}{\cos\alpha} = \frac{U\cos\beta}{\cos\alpha}$		
$V = \frac{U\cos\beta}{\cos\alpha} \quad \cdots (1)$ $\qquad VT\sin\alpha = h + UT\sin\beta \quad \cdots (2)$		
Sub (1) into (2)	1	Sub (1) into (2) and Multiply through by $\cos\alpha$
$\frac{U\cos\beta}{\cos\alpha}(T\sin\alpha) = h + UT\sin\beta \quad (\times\cos\alpha)$		
$UT\sin\alpha\cos\beta = h\cos\alpha + UT\cos\alpha\sin\beta$	1	Use identity and complete proof
$UT(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = h\cos\alpha$		
$UT\sin(\alpha-\beta) = h\cos\alpha$		
$T = \frac{h\cos\alpha}{U\sin(\alpha-\beta)}$		

MATHEMATICS EXTENSION 1 – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
METHOD 2 Collide at $t=T$		
$VT\cos\alpha = UT\cos\beta$ $\cancel{-\frac{1}{2}gT^2} + VT\sin\alpha = h \cancel{-\frac{1}{2}gT^2} + UT\sin\beta$	1	Equate x and y
$\therefore V\cos\alpha = U\cos\beta \cdots (1)$		components.
$VT\sin\alpha = h + UT\sin\beta$ $\times\cos\alpha$		
$VT\sin\alpha\cos\alpha = h\cos\alpha + UT\sin\beta\cos\alpha$		
Sub (1) into $T\sin\alpha \times U\cos\beta = h\cos\alpha + UT\sin\beta\cos\alpha$	1	Multiply through
$UT\sin\alpha\cos\beta - UT\cos\alpha\sin\beta = h\cos\alpha$		by $\cos\alpha$
$UT(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = h\cos\alpha$		
$UT\sin(\alpha-\beta) = h\cos\alpha$	1	replace $V\cos\alpha = U\cos\beta$.
$TU\sin(\alpha-\beta) = h\cos\alpha$		and complete proof
$T = \dfrac{h\cos\alpha}{U\sin(\alpha-\beta)}$		